B meson form factors from HQET simulations*

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We use simulations of heavy quark effective field theory to calculate the Isgur-Wise function, and we demonstrate the feasibility of calculating the matrix element for the $B \to \pi + l^+ \nu_l$ decay in the lattice heavy quark effective theory (HQET).

1. INTRODUCTION

We describe the calculation of the hadronic matrix elements that are required for the extraction of the V_{cb} and V_{ub} CKM matrix elements from experimental data [1]. To reach the bottom quark mass our strategy is to interpolate between results from relativistic quarks with $m_q \leq m_c$ and results from lattice HQET [2]. Here we discuss only the HQET simulations, as our clover form factor simulations have only just started.

All of our simulations use $n_f = 2$ dynamical staggered configurations with a volume $16^3 \times 48$ and $\beta = 5.445$.

2. ISGUR-WISE FUNCTION

The Isgur-Wise function is the QCD matrix element required in the extraction of V_{cb} from experimental data. Experimental measurements of the slope of the Isgur-Wise function vary from 0.31 to 1.17, and the variations in theoretical predictions are nearly as large [3]. Initial attempts to calculate the Isgur-Wise function in lattice HQET had problems either with the signal to noise ratio [4] or the renormalization factors [7]. The first complete calculation has been done recently by the Kentucky group [8].

We use the same method as the Kentucky group (see also [4,7]). We ran at all permutations of the following velocities: $(0,0,0)$, $(0.1,0,0)$, $(0.25,0,0)$ and $(0.5,0,0)$. Our sample size is 80 configurations, and our Wilson κ values are 0.160 and 0.163. A relative smearing function of $e^{-0.67r}$ was used between the quarks in the B meson. In Fig. 1 we plot the bare Isgur-Wise function for various time separations between the current and the B source. If the ground state has been isolated, then the Isgur-Wise function should be independent of this separation. The data for $\Delta t = 2, 3, 4$ are consistent within present errors.

It is traditional to report the slope of the Isgur-Wise function as a function of the dot product of

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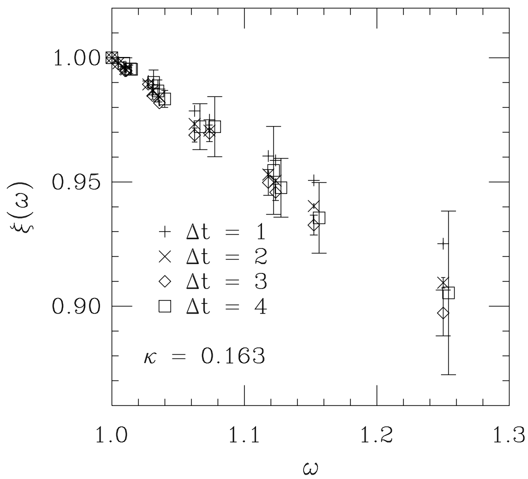

Figure 1. Unrenormalized Isgur-Wise function

Δt	$\kappa = 0.160$		$\kappa = 0.163$	
	ρ^2	χ^2/df	ρ^2	χ^2/df
2	0.415(8)	200/44	0.412(9)	171/44
3	0.48(4)	31/44	0.47(4)	24/44
4	0.48(12)	18/44	0.48(13)	17/44

Table 1
Preliminary fits to bare Isgur-Wise function data

the two meson velocities ($\omega = v \cdot u$). In Table 1 we report fits to

$$\xi(\omega) = 1 - \rho^2(\omega - 1) \tag{1}$$

Assuming negligible quark mass dependence, our best estimate is therefore $\rho^2 = 0.48(13)$ at the physical light quark mass. (We stress that it can not be compared with the experimental value until the renormalization factors calculated in [8] are included.) For comparison, at $\beta = 6.0$ the Kentucky group gets a bare $\rho^2 = 0.56$ and Hashimto and Matsufuru [7] get $\rho^2 \sim 0.54$ (where we have approximately removed the effect of tadpole improvement from their result [8]).

All the simulations of lattice HQET [7,8] show a very weak dependence of the slope on the light quark mass. However, the UKQCD collaboration [9] found a statistically significant decrease in ρ^2 with light quark mass in their simulations that used clover quarks for the b quark. We also tried fitting our bare Isgur-Wise function data to a fit model that had quadratic corrections of ω in Eq. 1. Acceptable fits were found with approximately the same slope as in Table 1 and positive curvature.

The velocity of the lattice HQET action is renormalized [10] because the action breaks Lorentz symmetry. As we described last year [11], we have tried to estimate the renormalized velocity from the dispersion relation of an HQET meson at finite residual momentum [7]. The renormalized velocity can be implicitly defined from

$$E(\underline{p}, v^R) - E(0, v^R) = \frac{\underline{v}^R.\underline{p}}{v_0^R} \tag{2}$$

where $E(\underline{p}, v^R)$ is the energy of the HQET meson at finite residual momentum ($\underline{p}$). In Table 2 we show the results of the non-perturbative velocity renormalization. We fit all the correlators in the time region 5 to 13 and obtained correlated χ^2/df slightly less than one. For comparison, we also show the results for the perturbative renormalization calculated by Mandula and Ogilvie [5,6], using both a boosted (g^2/u_0^4) PT_{boost} and bare coupling PT_{bare}, as well as using the tree-level tadpole improved estimate [8].

Vel	NP	PT_{bare}	PT_{boost}	tadpole
0.1	0.04(2)	0.074	0.051	0.085
0.25	0.13(2)	0.18	0.12	0.21
0.5	0.25(3)	0.34	0.23	0.41

Table 2
Various estimates of the HQET velocity renormalization

The results in Table 2 show that the velocity renormalization is large. These results would suggest that perturbation theory with a boosted coupling agrees best with the non-perturbative result. However, other analyses found better agreement between the tree-level tadpole scheme [8] and the non-perturbative calculations [5–7]. This issue is under investigation.

3. $B \to \pi$ FORM FACTOR

The observation of the decay $B \to \pi + l^+\nu_l$ allows a determination of V_{ub}, if the relevant QCD form factors can be calculated. There have been a number of lattice QCD calculations of the required form factors (see [1,12] for reviews). However, previous approaches suffer from the drawback that calculations are done at large q^2, thus requiring a large extrapolation to $q^2 \approx 0$, where measurements are currently made. To reach a low q^2 requires large meson velocities—not easily achieved for heavy mesons in the NRQCD- or propagating-quark-approaches. As we have shown [11], a good signal can be obtained for the HQET B meson with a large velocity ($v \approx 0.8$), so we propose the use of HQET—light simulations to explore lower q^2 (see [13] for similar ideas). It is not clear that HQET will be a good approximation to the dynamics of the B meson, at these values of q^2, nor that a sufficiently good signal will be obtained. However, results should be very useful in helping to reduce the heavy quark extrapolation errors over simulations that only use clover quarks.

Because $B \to$ light meson form factors have never been studied before using lattice HQET (although the static limit was studied in [14]), we have computed the matrix element for $B \to \pi + l^+\nu_l$ using HQET to check for a signal. We use the setup described in [14] with the heavy clover quark replaced by a HQET quark. In Fig. 2 we plot the ratio of three point functions to two point functions

$$\frac{\langle C_3(t;t_f)\rangle}{\langle C_2(t)_L\rangle\langle C_2(t_f-t)_B\rangle} \tag{3}$$

that is proportional to the $\langle B \mid J_\mu \mid \pi\rangle$ matrix element, as a function of the operator time t. The B source is fixed at $t_f = 23$, and the light meson source is fixed at $t = 0$.

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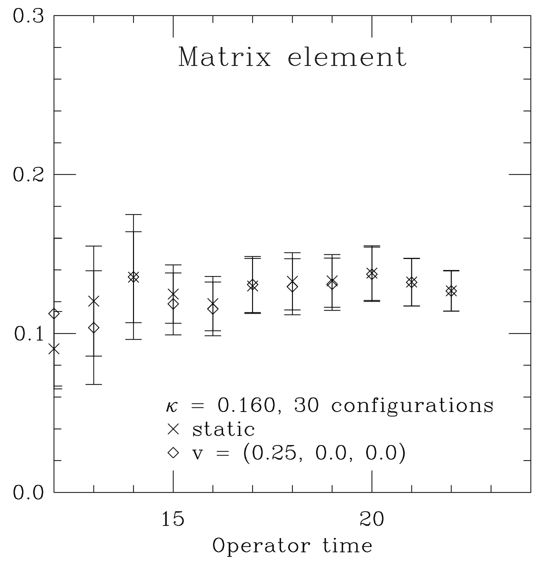

Figure 2. HQET—light matrix element

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